Supplementary text

Quantifying the Structural Impact of Random Vaccination: In general, random and health-based vaccination programs immunize individuals without considering connectivity (Anderson & May 1991). Therefore, the removal of nodes is random with respect to degree. It follows that the mean original degree on the residual network following random vaccination ($\langle k \rangle_v$) is simply equal to the mean degree in the original network, $\langle k \rangle_v = \langle k \rangle$. We next derive the mean residual degree on the vaccinated network ($\langle k_r \rangle_v$) following the logic of the derivation of $\langle k_r \rangle_r$ above. Consider a network of size N. If a fraction of the network, π, is vaccinated at random, then the number of nodes in the vaccinated portion is $N\pi$ and the number stubs attached to unvaccinated nodes is $N\sum kp_k(1-\pi) = N\langle k \rangle(1-\pi)$. Dividing this quantity by the total number of stubs in the original network, $N\langle k \rangle$, we find that the fraction of stubs attached to unvaccinated nodes is equal to $(1-\pi)$. As above, we assume random connectivity in the network, and square this quantity to find the fraction of stubs that lie on edges connecting two unvaccinated nodes. We then multiply by the total number of stubs in the original network and divide by the number unvaccinated nodes to compute the average degree of an unvaccinated node, which yields

$$\langle k_r \rangle_v = (1-\pi)\langle k \rangle . \qquad \textbf{[S1]}$$

We model vaccination at coverage levels comparable to those attained by epidemic immunization by setting π equal to the expected size of the epidemic,

$S = 1 - \sum p_k (1 + (u-1)T)^k$ as derived in (Newman 2002), with u as defined in the text and is the probability of transmission across an edge.

Analytically, we find that the difference between the residual degree on the naturally immunized networks and on the vaccinated networks is small (<1) in general. The two processes yield virtually identical results on the Poisson network. Small-world networks seem to be better protected by random vaccination than natural immunization ($\langle k_r \rangle_v < \langle k_r \rangle_r$) whereas the reverse is true for scale free networks ($\langle k_r \rangle_r < \langle k_r \rangle_v$). In both cases, the differences between the two forms of immunization decreases as infectivity (β) increases (Figure Supplementary 1).

<u>Estimates for Average Residual Degree:</u>

Our estimate for the average residual degree in the residual network after an epidemic is given by:

$$\langle k_r \rangle_r = \frac{\left(\sum k p_k (1-\omega_k)\right)^2}{\sum p_k (k - 2v_k) \sum p_k (1 - v_k)}$$

where $u = \dfrac{\sum_k k p_k \left(1 + (u-1)T\right)^{k-1}}{\sum_k k p_k}$, $v_k = 1 - \left(1 - T + Tu\right)^k$ and $\omega_k = 1 - \left(1 - T + Tu\right)^{k-1}$.

Solving for the mean residual degree using the residual degree distribution given by Eq (9) of Newman 2005 gives:

$$\langle k_r \rangle_{r_N} = \frac{\left(\sum k p_k u_N^{k-1}\right)^2}{\sum k p_k \sum p_k u_N^k} \qquad \textbf{[S2]}$$

where u_N is the solution to $u_N = 1 - T + T\frac{\sum kp_k u_N{}^{k-1}}{\sum kp_k}$, and gives the mean probability that a vertex is not infected by a specified neighbor. We can express u_N in terms of u as $u_N = 1 - T + Tu$ and re-express our equation for $\langle k_r \rangle_r$ as

$$\langle k_r \rangle_r = \frac{\left(\sum kp_k u_N^{k-1}\right)^2}{\left[\sum kp_k - 2p_k\left(1 - u_N^k\right)\right]\left[\sum p_k u_N^k\right]}. \qquad \textbf{[S3]}$$

A comparison of Equations [S2] and [S3] shows that the two estimates for the mean residual degree are very similar. Our estimate ($\langle k_r \rangle_r$) differs in that it excludes edges that have been conduits for disease transmission from the calculation (there are $Np_k\left(1 - u_N^k\right)$ of these edges, where N is the size of the network). In Supplementary Figure 2, we show that our estimate for average residual degree performs slightly better when compared to simulated epidemics on Poisson and scale-free networks than the estimate that follows from Newman (2005.)

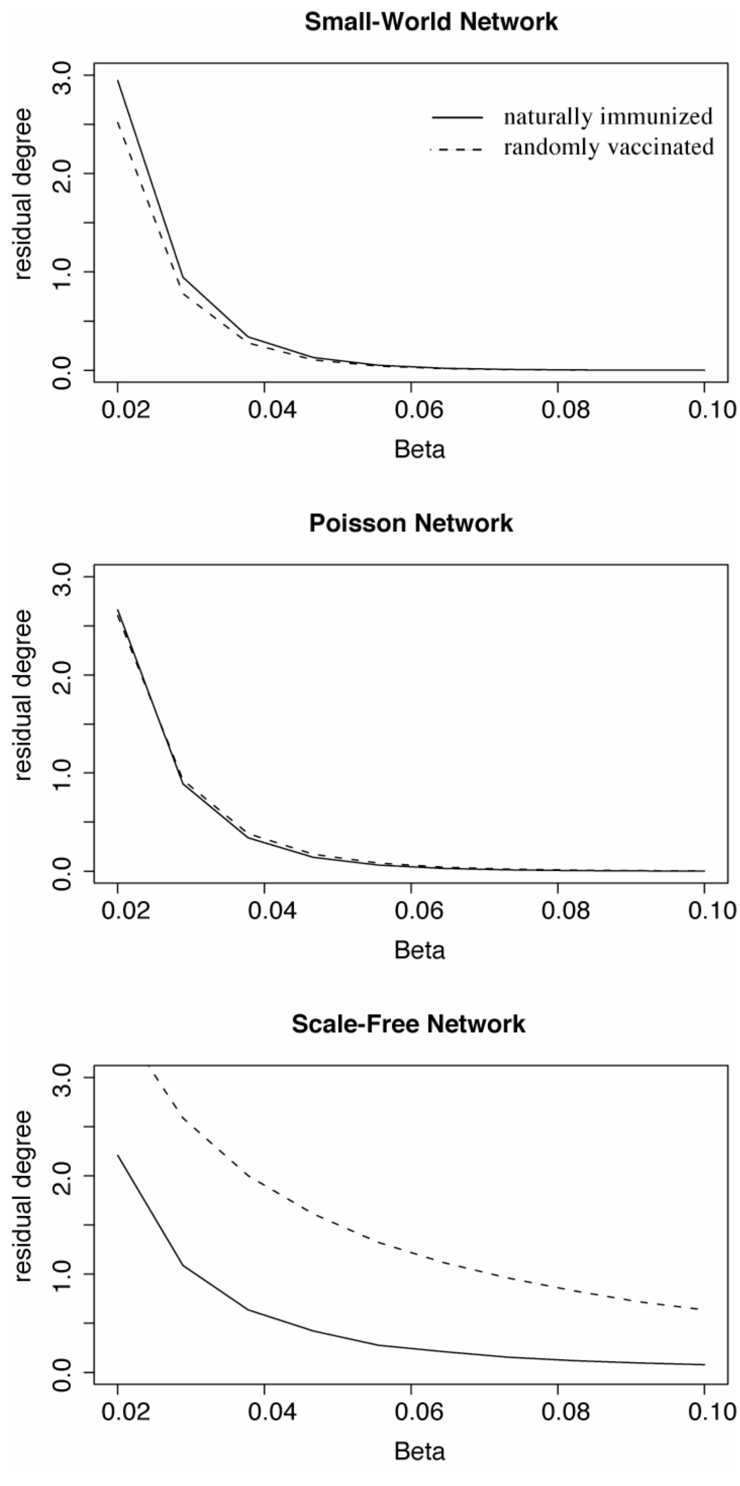

Figure Supplementary 1. Residual degree on residual network, $\langle k_r \rangle_r$, for naturally immunized (Equation 2) and randomly vaccinated networks (Equation 5).

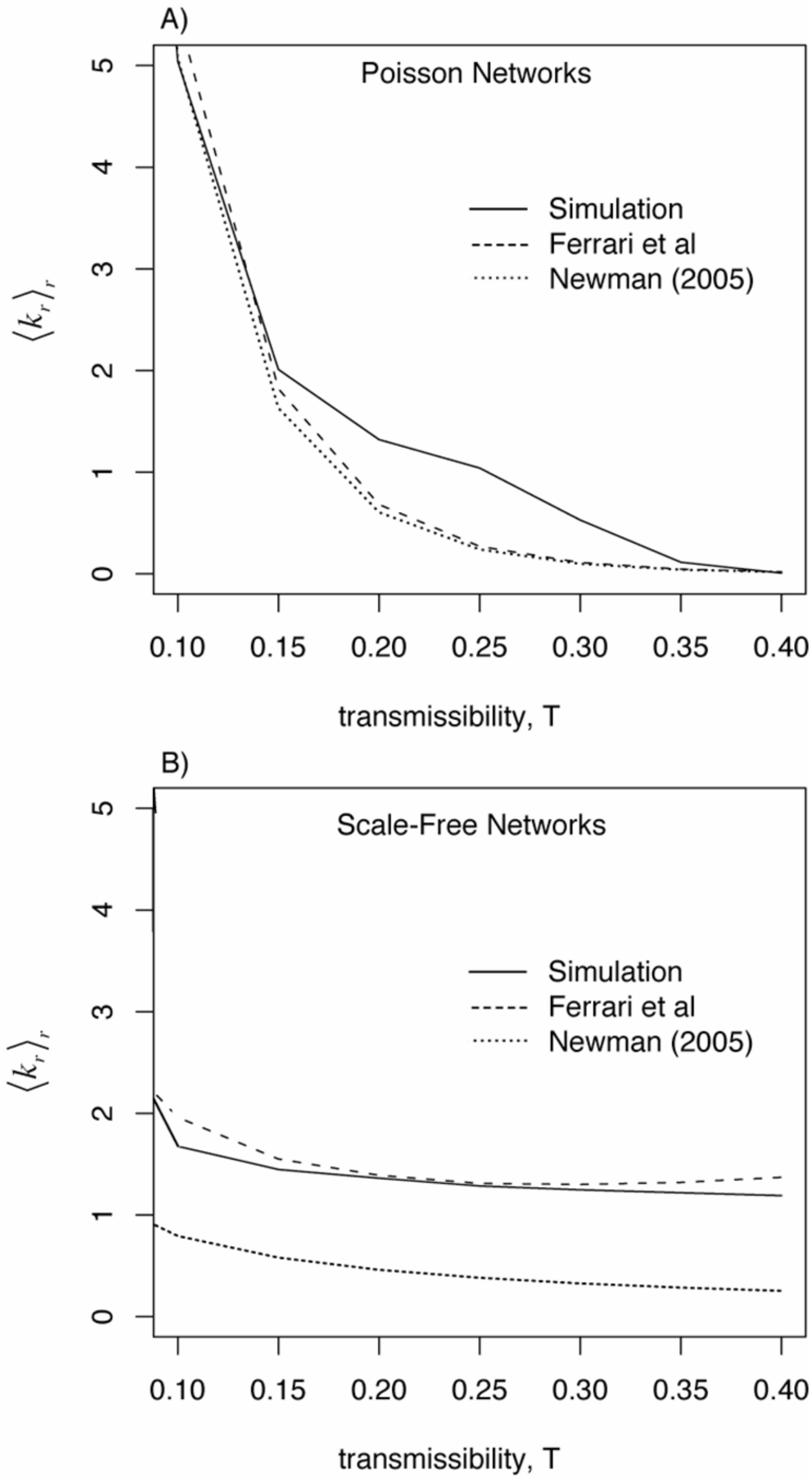

Figure Supplementary 2: Average residual degree estimates for A) Poisson and B) scale-free using 1) the method described above, and 2) the method form Newman (2005) compared to results from simulated networks.