Supplementary material for

Travelling waves and spatial hierarchies in measles epidemics. Nature 414: 716-723

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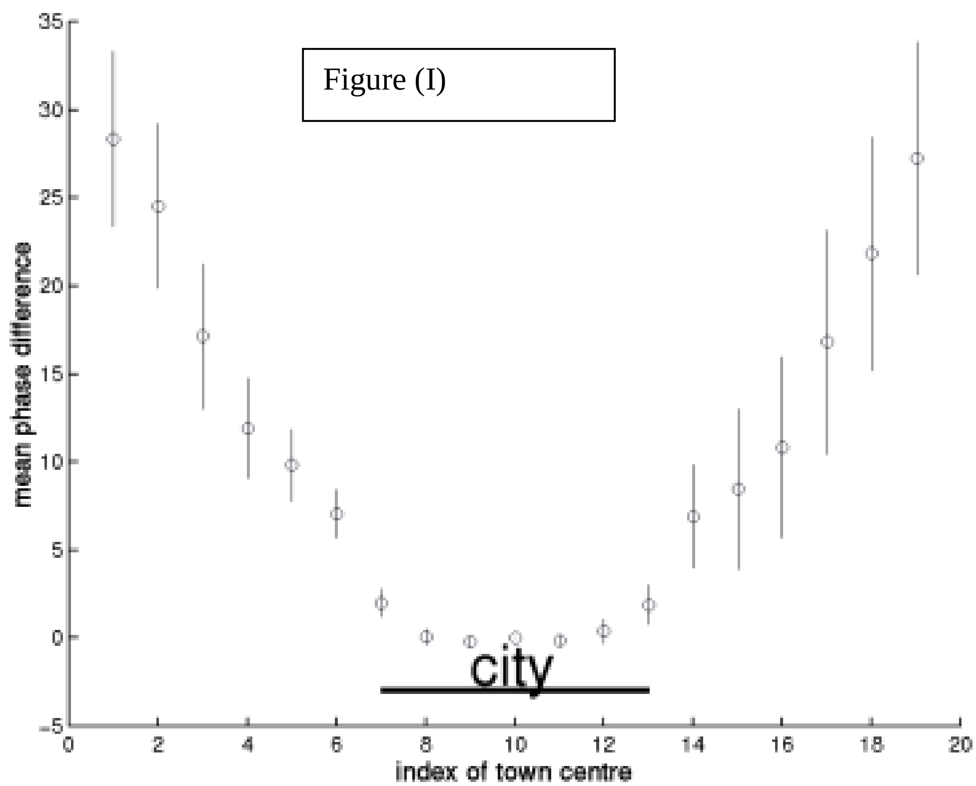
Figure (I) (which accompanies Box 2, figure a) shows the mean (+/s.e.) biennial wavelet phase of 60 simulations of the spatial TSIR model with small centres, defined in Box 2 of the paper. To explore the phase relationships arising from approximately phase-locked biennial cycles, the simulations were run for 10 years, starting from the phase-locked deterministic attractor of the system.



Notes

1. The phase lag is not an 'edge' effect, since wrapped boundary condition also generate a phase lag near the city.
2. As we move away from the city, note that the phase error bars widen. This is partly due to greater irregularity nearer the edge, but also partly because occasional simulations drop onto the opposite attractor (with 180^o phase lag). As noted in Box 2, this partly reflects distance from the city, but also a genuine edge

effect (note that Norwich and its environs are near the coast, hence an ‘edge’ in terms of measles transmission).

3. Previous work (Bjørnstad *et al*. 2001, *op. cit.*) shows that the relative measles transmission rate (R_o) in the England and Wales data set is constant across 3 orders of magnitude of population size. In fact, the crude models used here show slight increases in transmission rate with coupling (because tight coupling in the crude way we have defined it can increase R_o).
   This is illustrated in Figure (II), which presents a phase analysis of the case when large centres are coupled (cf Box 2, figure (b) in the paper). Increased coupling between boroughs increases their effective R_o and causes them to lead the ‘periphery’ slightly, by 1 time step (2 weeks) – note also that the phase analysis picks up this small phase lag more sensitively than the crude plot of average biennia in Box 2.
   This effect – which is a byproduct of our coupling model, cannot cause the progressive phase lag of the periphery seen in Figure (I); this latter is driven by ‘sparks’ of infection in the hierarchical system, as described in Box 2. However, the effect of coupling on effective transmission does complicate the analysis of the relationship between waves and coupling. We are currently exploring this issue: preliminary analyses indicate that hierarchical waves will occur at a range of ‘intermediate coupling rates, as long as the overall dynamics are biennial and major epidemics occur in the same year.

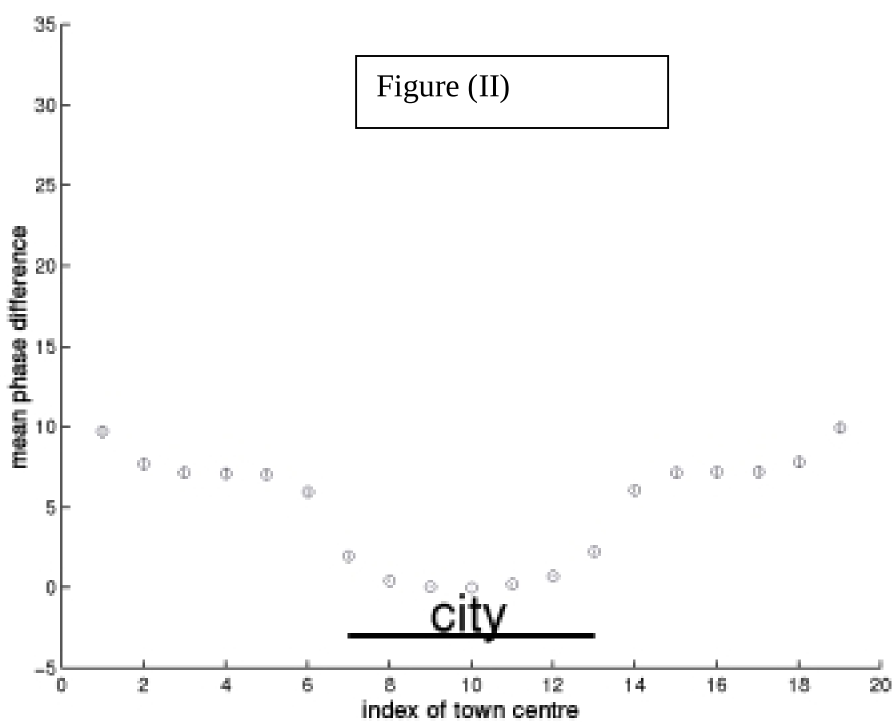


Figure (II)