

Supplementary Information

Table S1. List of parameters

Symbol	Interpretation	Units	Estimated or data?
σ	Rate of loss of immunity	1/wk	estimated
β	Seasonal transmission rates	1/wk	estimated
κ	Boosting coefficient	–	estimated
γ	Recovery rate	1/wk	estimated
β_{SD}	Environmental stochasticity	–	estimated
τ	Overdispersion in reporting	–	estimated
ρ	Observation probability	–	estimated
c	Reported cases	–	data
b	Births	–	data

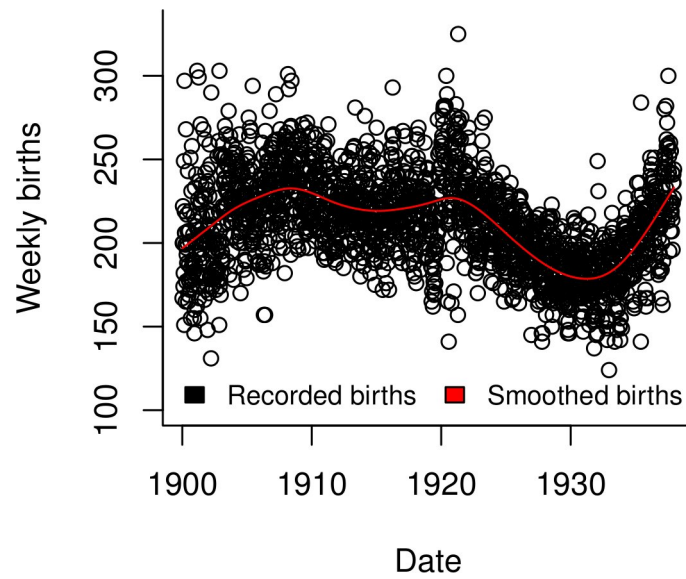


Figure S1. Births. The black dots show the recorded number of weekly births, and the red (gray) line the smoothed births, which were used as the mean of Poisson draws for weekly births in the model.

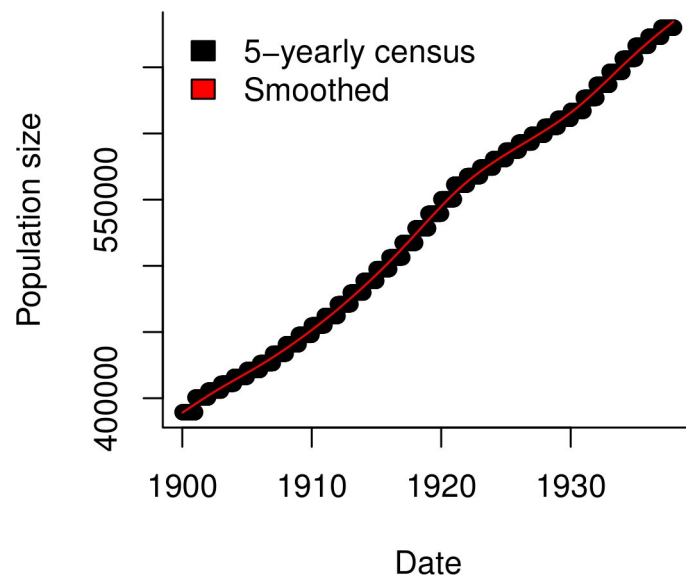


Figure S2. Population size. The black dots show the censused population size, taken every five years. The red line shows the smoothed population size with which the model was forced.

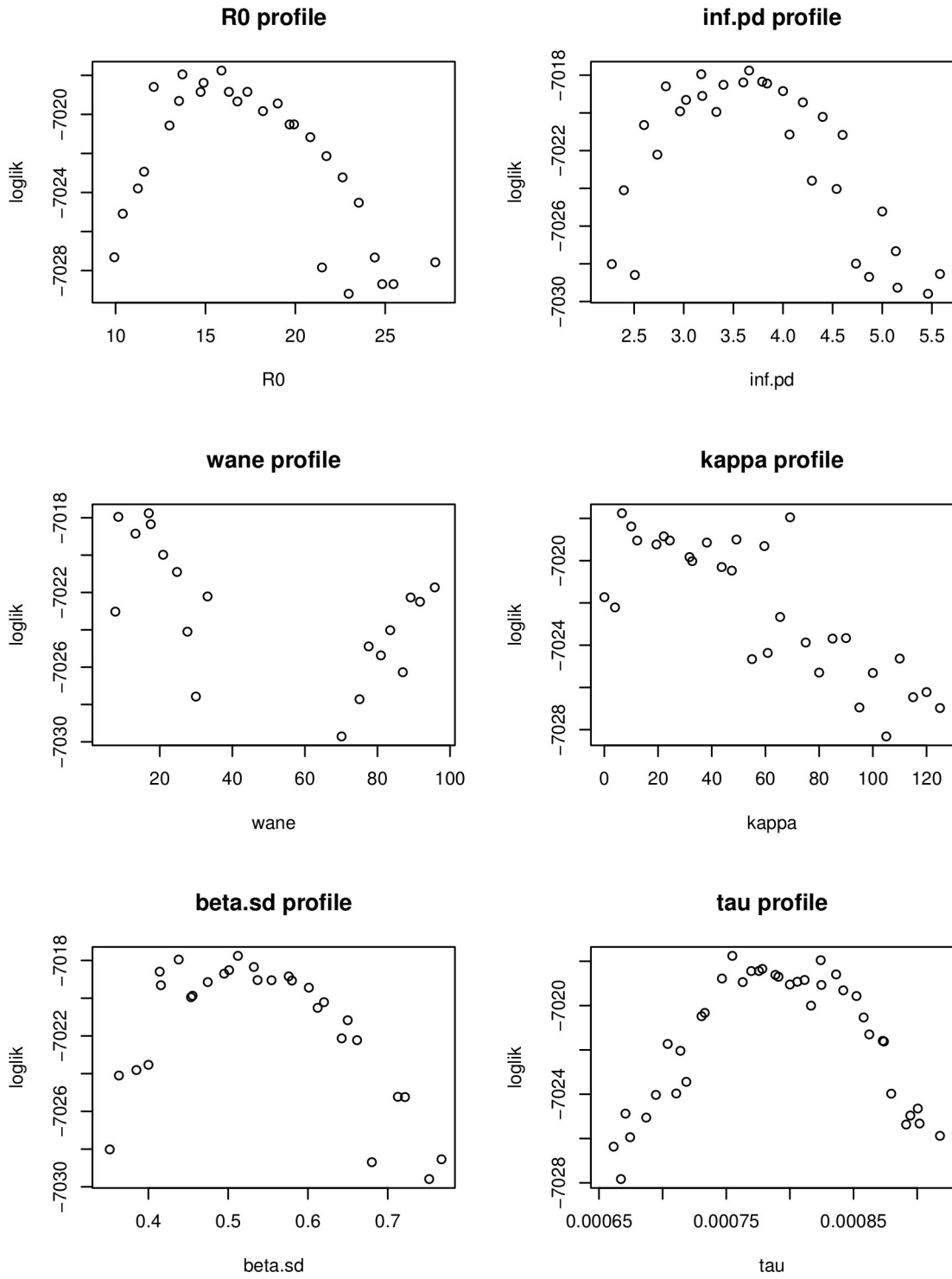


Figure S3. Likelihood profiles for estimated parameters.

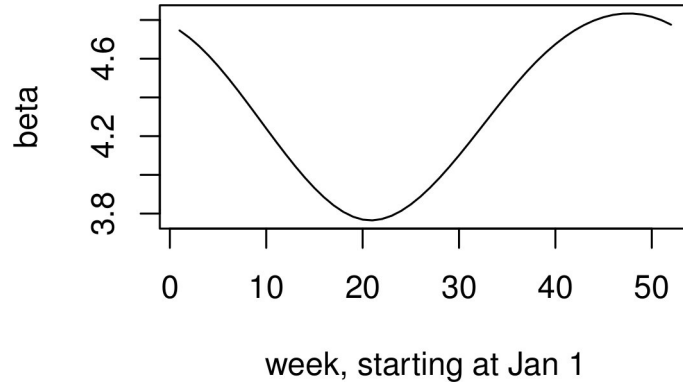


Figure S4. Estimated seasonality, computed from the estimates of the three β s and the B-spline basis functions.

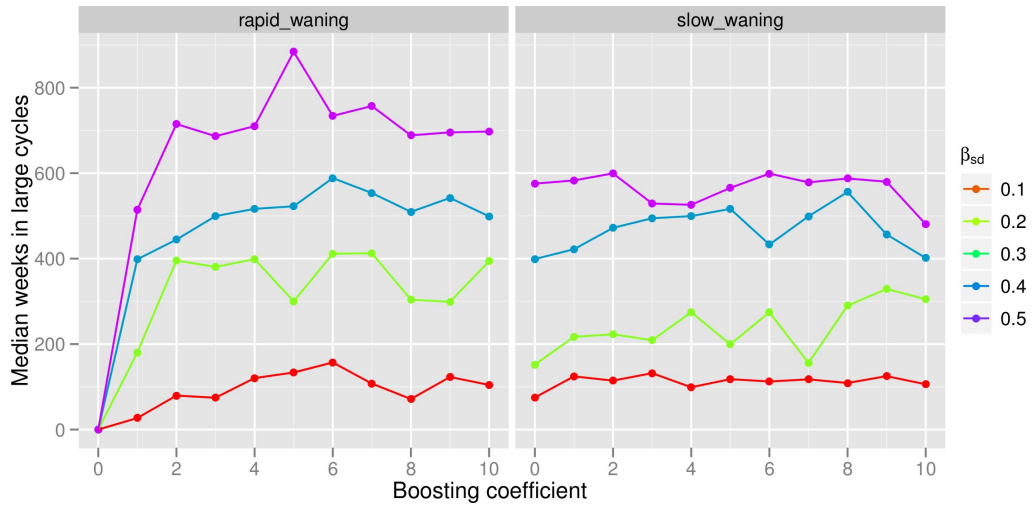


Figure S5. Effect of κ and β_{sd} on the time spent in multiannual cycles. Parameters used for rapid waning simulations: $\beta_1 = 5.0$, $\beta_2 = 3.2$, $\beta_3 = 4.8$, $\gamma = 0.27$, $\sigma = 1.1 \times 10^{-3}$, $\tau = 7.5 \times 10^{-4}$, $\mu = 1.3 \times 10^{-4}$, $\rho = 0.15$, $S_0 = 0.049$, $I_0 = 7.7 \times 10^{-4}$, $R_0 = 0.95$, $W_0 = 2.9 \times 10^{-3}$. Parameters used for slow loss of immunity simulations: $\beta_1 = 5.2$, $\beta_2 = 3.4$, $\beta_3 = 5.0$, $\gamma = 0.24$, $\sigma = 2.2 \times 10^{-4}$, $\tau = 7.7 \times 10^{-4}$, $\mu = 1.3 \times 10^{-4}$, $\rho = 0.15$, $S_0 = 0.043$, $I_0 = 8.7 \times 10^{-4}$, $R_0 = 0.95$, $W_0 = 0.0029$

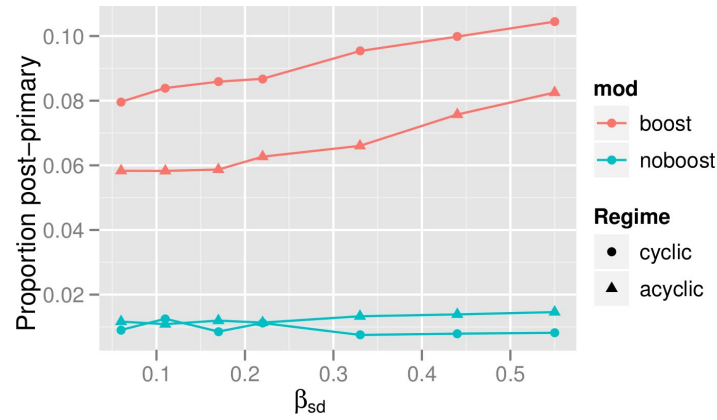


Figure S6. Effects of stochasticity on the proportion of cases that are post-primary during the cyclic regime (circles) and acyclic regime (triangles) for stochastic models with rapid (pink lines) vs slow (blue lines) loss of immunity, and the boosting coefficient, $\kappa = 7$. Parameters used for rapid waning simulations: $\beta_1 = 5.0$, $\beta_2 = 3.2$, $\beta_3 = 4.8$, $\gamma = 0.27$, $\sigma = 1.1 \times 10^{-3}$, $\tau = 7.5 \times 10^{-4}$, $\mu = 1.3 \times 10^{-4}$, $\rho = 0.15$, $S_0 = 0.049$, $I_0 = 7.7 \times 10^{-4}$, $R_0 = 0.95$, $W_0 = 2.9 \times 10^{-3}$. Parameters used for slow loss of immunity simulations: $\beta_1 = 5.2$, $\beta_2 = 3.4$, $\beta_3 = 5.0$, $\gamma = 0.24$, $\sigma = 2.2 \times 10^{-4}$, $\tau = 7.7 \times 10^{-4}$, $\mu = 1.3 \times 10^{-4}$, $\rho = 0.15$, $S_0 = 0.043$, $I_0 = 8.7 \times 10^{-4}$, $R_0 = 0.95$, $W_0 = 0.0029$

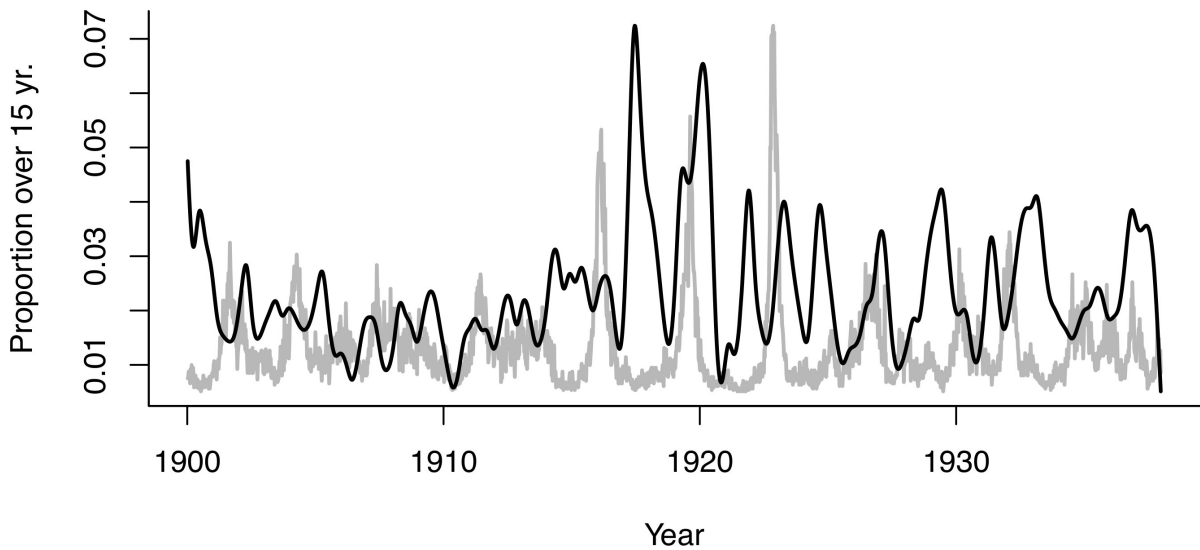


Figure S7. The black line shows the smoothed proportion of cases in people over the age of 15 years. The gray line, which shows the fluctuation in incidence as shown in figure 1 of the main text, is included to visually highlight the relationship between the highly cyclic regime and the increased proportion of cases in people over the age of 15 years.

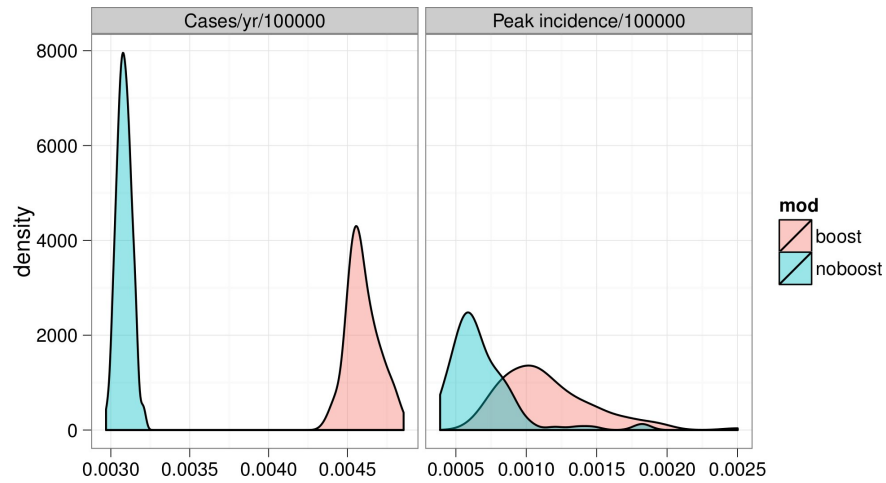


Figure S8. Kernel density estimates of distributions of (a) total annual incidence per 100,000 and (b) peak case reports per week per 100,000 in the vaccine era. Calculated from same simulation data as figure S9

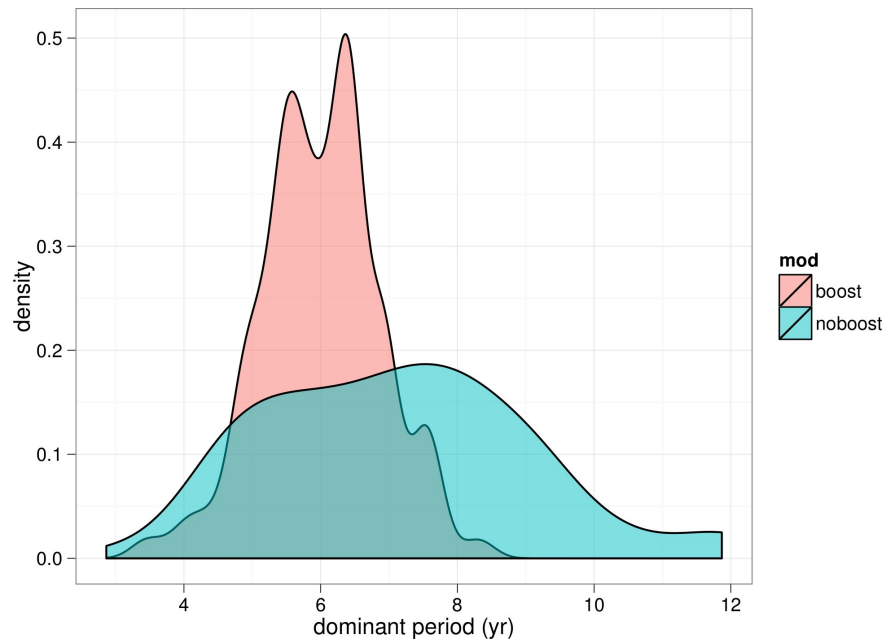


Figure S9. Distribution of dominant periods in 100 simulations from each model, calculated from 80 yr of simulations, starting 20 yr after the onset of vaccination.

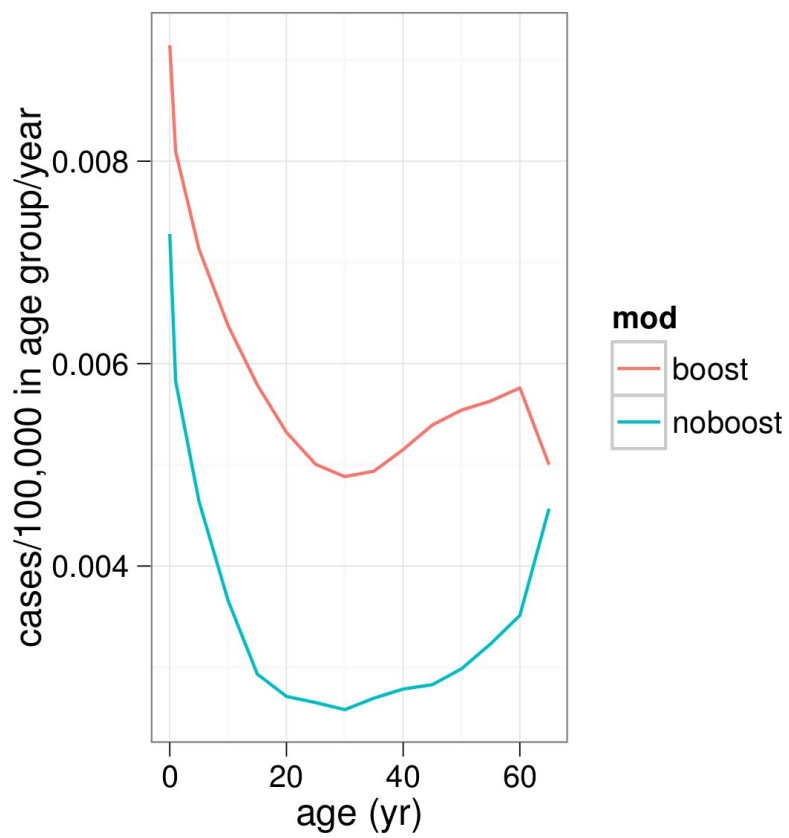


Figure S10. Annual incidence per 100,000 people in each age group, calculated from same simulations as figure S9

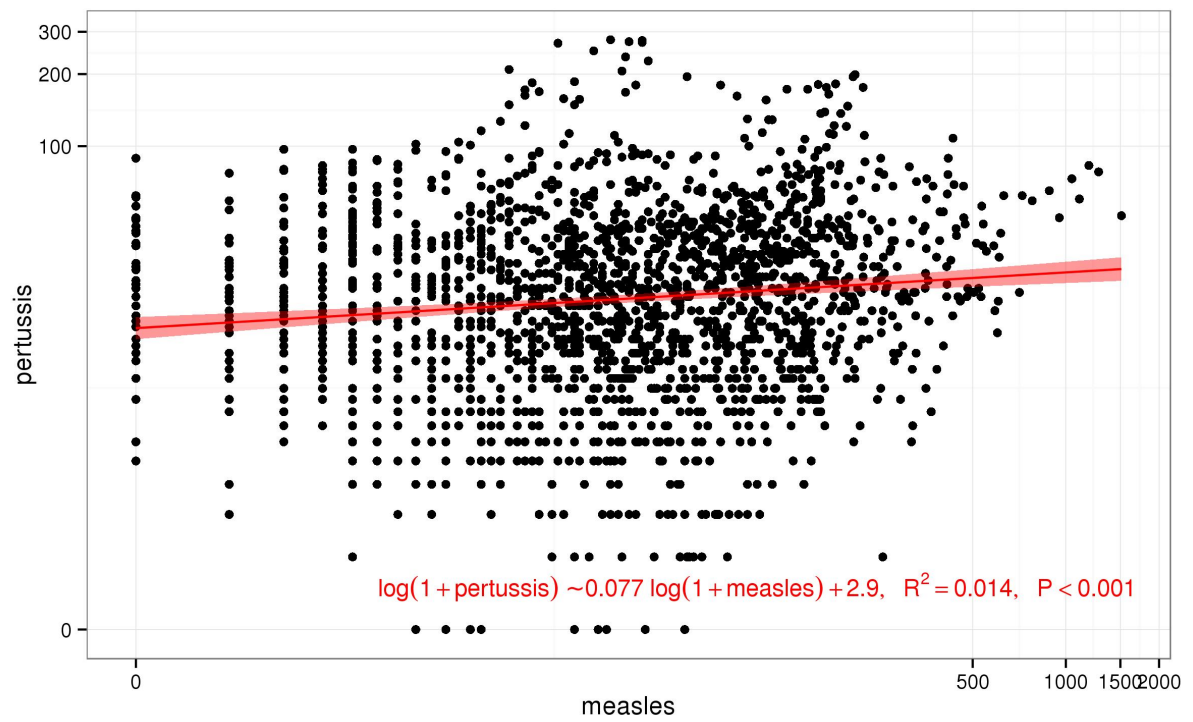


Figure S11. The correlation between measles and pertussis cases is weak and statistically significantly positive. Were ecological interference occurring between these infections, one might expect to see a negative correlation [33].

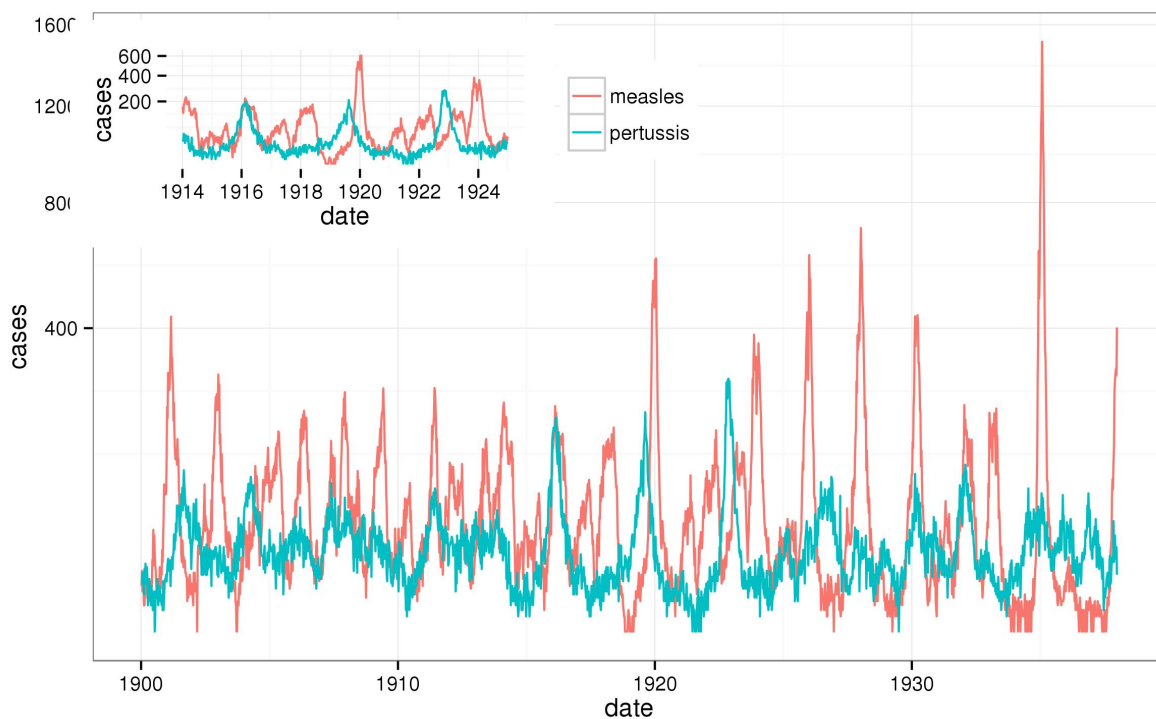


Figure S12. Time series of measles and pertussis in Copenhagen. The inset shows a detail of the 1914–1924 period. No clear pattern indicative of ecological interference between these infections is evident.